

Superconducting Quantum Interference in Fractal Percolation Films. Problem of $1/f$ Noise

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Abstract

An oscillatory magnetic field dependence of the DC voltage is observed when a low-frequency current flows through superconducting Sn-Ge thin-film composites near the percolation threshold. The paper also studies the experimental realisations of temporal voltage fluctuations in these films. Both the structure of the voltage oscillations against the magnetic field and the time series of the electric "noise" possess a fractal pattern. With the help of the fractal analysis procedure, the fluctuations observed have been shown to be neither a noise with a large number of degrees of freedom, nor the realisations of a well defined dynamic system. On the contrary the model of voltage oscillations induced by the weak fluctuations of a magnetic field of arbitrary nature gives the most appropriate description of the phenomenon observed. The imaging function of such a transformation possesses a fractal nature, thus leading to power-law spectra of voltage fluctuations even for the simplest types of magnetic fluctuations including the monochromatic ones. Thus, the paper suggests a new universal mechanism of a "1/f noise" origin. It consists in a passive transformation of any natural fluctuations with a fractal-type transformation function.

1 Introduction

Studies of the object critical behavior near the superconducting phase transition or a metal-dielectric transition are related to the fundamental problems of describing systems characterized by a scale or time invariance, or so-called fractal (self-similar) systems [1].

The $1/f$ noise, or the "flicker noise", remains a very important and enigmatic problem which is far from its solution. The fact that a noise of the $1/f$ type is found practically everywhere (in physical, biological and even in social systems) stimulates the search of some general approaches to this problem. The language of fractals is just such an approach, and a number of theoretical works on the "flicker noise" have been written in this language. In particular, there appeared a theory of self-organized criticality [2] in which it was shown that a dissipative dynamic system evolves naturally to a critical state where characteristic time and length scales are absent. Such states possess a scale-invariant (fractal) structure and undergo fluctuations with a spectrum of the $1/f$ type.

There are not enough experimental works carried out in this field, because of the difficulties of obtaining model fractal systems and of observing sufficiently distinct

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fluctuations of physical quantities in them. In this connection, quantum interference effects in macroscopically inhomogeneous media look rather promising.

An AC current passing through systems with weak bonds in an applied magnetic field H at a temperature below the superconducting transition temperature is known to induce a DC voltage V_{DC} oscillating against a variation of the field H [3]. Such oscillatory dependences $V_{DC}(H)$ were observed by Yurchenko et al. [4] in Nb powders, and in the Clarke drop-type interferometers [5]. The dependences were attributed to the quantum interference effects in asymmetrical contours with a weak coupling, i.e. the magnetic flux quantization induces the critical current oscillations and the respective voltage oscillations. According to De Waele and De Bruyn Ouboter [3], when an AC current passes through a system of two superconductors weakly connected by an asymmetric double-point contact with an amplitude larger than the critical current of the junction, then a DC voltage is observed even in the absence of the DC current. This rectification process takes place due to the fact that the absolute values of the voltage oscillations are shifted with respect to each other for different signs of a DC current. The resultant DC voltage depends periodically on the applied magnetic field with a period $\Delta B_{\perp} = h/2eO$ (O being the area enclosed between the two contacts) and it is antisymmetric under field reversal. A superconducting granulated film at the percolation threshold can be obviously considered as a random set of asymmetric Josephson contours. In this connection, as observed by Gerber and Deutscher [6], DC voltage oscillations in granulated Pb and Al percolation films could be explained by the superconducting quantum interference.

Thus, the DC voltage oscillations in percolation films have been associated with the topological structure of quantizing contours; however, nobody has ever tried to relate them to the sample fractal structure. Thus, the idea has originated to investigate correlations between the structure of quantum voltage oscillations in a magnetic field and topological properties of thin-film superconducting composites in the vicinity of the percolation threshold, within the framework of the fractal approach, in order to elucidate the mechanism of the “1/f noise” origin in self-similar objects.

Thus, two curious phenomena, observed in granulated percolation thin film Sn-Ge systems, form the object of our investigations:

1. Fractal character of voltage V_{DC} induced by an AC current passing through system as a function of the external magnetic field H - $V_{DC}(H)$.
2. Time fluctuations of the voltage at a constant value of the magnetic field H .

2 Experiment

Granulated films for the Sn - amorphous Ge system with monotonically varying structural characteristics are made by vacuum condensation of Sn on a long (60mm) substrate along which a temperature gradient is created. The temperature at the midpoint of the substrate (80°C) corresponds to the temperature of the Sn condensation mechanism change. Sn is deposited on the previously prepared Ge layer 50nm thick. The effective thickness of the Sn layer is 60nm. The metallic condensate is covered with amorphous Ge from the top too.

On using this technique granulated films are obtained. The films have a varying (along the substrate) structure: near the “cool” end they have a labyrinthine structure with low resistance (1Ω per square), while at the “hot” end they have an island structure with high resistance (80kΩ per square). This structural change results in the metal-dielectric transition within one series, consisting of 30 samples, the percolation threshold being near the substrate midpoint. The technique of obtaining granulated films and their resistive properties on the metal and dielectric

Figure 1: Electron micrograph of Sn-Ge sample near the percolation threshold. Light regions correspond to metal.

sides of the metal-nonmetal transition are described in detail in Refs [7, 8]. For the present investigations, we chose the films near the percolation threshold with a characteristic structure depicted in Fig.1.

To achieve our aim we carried out experiments of two types:

- (a) registration of rectified voltage as a function of an external magnetic field while passing AC through the sample, $V_{DC}(H)$;
- (b) recording the low-frequency part of “noises” in the sample while passing DC through it in the absence of an external magnetic field, $V(t)$.

The electric measurements were carried out according to the standard four-probe technique. The DC was measured to an accuracy of 0.01% in the range of 0.01 - 2mA for various samples, and AC of 1 - 200kHz frequency, 0.2 - 2mA effective value and 0.1% stability was provided by a generator of sinusoidal signals through a decoupling transformer. The voltage in the both experiments was registered by a digital voltmeter with the measuring frequency amounting to 12 measurements per second and the accuracy of 0.1μV. The usual length of recording a noise dependence was 8192 points. Input and processing of experimental data were made with the help of the original computer algorithms.

Samples were immersed in liquid He inside a superconducting solenoid. The measuring cell construction enabled one to vary the sample orientation relative to the magnetic field direction. To avoid possible hysteresis phenomena, the dependence $V_{DC}(H)$ was recorded with various directions of the current through the solenoid. The scanning step in the magnetic field varied, and did not exceed $10^{-6}T$. In the operating temperature range 2.5 - 4.2K the temperature stabilization was not worse than $10^{-3}K$ with the superconducting transition width of 0.2 - 1.0K. To avoid a possible effect of temperature fluctuations, temperature values were simultaneously recorded with an accuracy of $2 \times 10^{-4}K$, but no noticeable drifts or fluctuations were found.

Figure 2: Oscillatory structure of voltage across a Sn-Ge sample for various angles θ between the applied magnetic field and the normal to the surface. $T = 3.354K$, $I_{AC} = 0.2mA$, $f = 100kHz$.

3 Results and discussion

3.1 Fractal analysis of the structure of superconducting quantum voltage oscillations

As it was described in our brief report [9], the samples under investigation were characterized by highly extended superconducting transitions. At the temperatures below the midpoint of the resistive transition, clearly manifested oscillations of the DC voltage V_{DC} were observed against the magnetic field H applied, when an AC current passed through the sample. The amplitude and frequency of the current did not affect the form of the $V_{DC}(H)$ dependence significantly. The results could be easily reproduced. Fig.2 shows the $V_{DC}(H)$ dependence for various orientations of the film relative to the magnetic field. The scale of the oscillatory structure in the field is inversely proportional to the cosine of the angle between the applied magnetic field and the normal to the sample plane. The emergence of the normal magnetic field component alone as well as the antisymmetry of the oscillatory structure relative to $H = 0$ indicate the quantum-interference origin of $V_{DC}(H)$.

A decrease in temperature leads to new oscillations in the case of strong fields (Fig.3), and the mean square deviation S of the rectified voltage which characterizes the average amplitude of oscillations, has a maximum near $2.6K$. Such a behavior of $V_{DC}(H)$ and S is associated with an increase in the number of quantizing contours and with an increase in their critical fields with decreasing temperature. A further decrease in temperature causes a decrease in S due to closure of the percolation net and a decrease in the number of weak bonds.

In the system under investigation, quantizing contours belong to the skeleton of the metal percolation cluster. A percolation system near the percolation threshold can be regarded as fractal, self-similar, on the scales smaller than the correlation length ξ , and as homogeneous on larger scales [10, 11]. In the homogeneous case, the dependence of the infinite cluster mass contained in a $a \times a$ square is given on this scale a by the expression $M \cdot a^d$, where d is the conventional topological dimension. When $a \ll \xi$, an anomalous behavior of mass is observed in the self-similar mode, $M \cdot a^D$. D is called the fractal dimension. Density, in its turn, behaves as

Figure 3: Magnetic field dependences of V_{DC} at various temperatures.

$$\rho = \frac{M}{a^d} = \begin{cases} a^{D-d} & \text{for } a < \xi, \\ \text{const} & \text{for } a > \xi, \end{cases} \quad (1)$$

i.e. on scales larger than ξ the density becomes constant. The theory predicts the relation $D = d - \beta/\nu$. Here β and ν are critical exponents of the infinite cluster density and the correlation length, respectively. In the 2D case $\beta = 0.14$, $\nu = 1.33$, so that $D = 1.896$.

To analyze the fractal dimensions of real objects, we processed electronic microscope pictures of the investigated Sn-Ge samples on a computer. The procedure is the following. First, we choose an initial point in the computer picture of the percolation cluster (Fig.1). Next, the masses M (i.e. quantities of white points) of the infinite cluster are counted; the counted points must lie in the square centered at the chosen initial point and $2a$ large, from 0 to the picture boundary. This operation is repeated several times for different position of the initial point on the infinite cluster. After averaging over all the initial points, the quantity $M(a)$ together with the density values $\rho(a) = M(a)/a^2$ is presented in the double logarithmic coordinates (Fig.4). The radius, along which the fractal behavior is replaced by the homogeneous one, corresponds to the percolation length of the correlation, and equals about 60 points for the sample considered. By the least-squares method, we find the slope of the dependence $\ln(M)$ on $\ln(a)$ in the fractal and homogeneous modes, which correspond to the fractal $D = 1.88$ and conventional $d = 2.005$ dimensions. One can see that the fractal dimension value agrees well with the theory.

In view of the fact that the scale of oscillations in a magnetic field under the conditions of quantum interference is unambiguously connected with the size of quantizing contours, we can assume that voltage oscillations occurring at a percolation fractal cluster have a fractal structure too. The fractal dimensions of the plots of the $V_{DC}(H)$ dependence can be calculated formally by covering them with cells of width bh along the magnetic field axis and of length bv along the voltage axis so that the minimum size of a cell is $h \times v$. Then the fractal dimension D_c over the covering is determined from the following dependence of the number N_c of cells required for covering the curve on the scale b of the cells: $N_c(b; v, h) \sim b^{-D_c}$ [12]. The minimum cell width is $h = 1$, and for the height v we can take a value of the order of resolution of the measuring voltmeter. We calculated the cell dimensions of the $V_{DC}(H)$ dependence at all temperatures. The dependence of the number N_c of cells

Figure 4: The dependences of the infinite cluster mass M and density ρ on linear size of sample under investigation (see Fig.1).

covering the $V_{DC}(H)$ curve on the cell scale b for a certain value of temperature is presented in Fig.5 in logarithmic coordinates. The fractal dimensions D_c obtained by approximating the experimental data were found to be 1.6 - 1.7. They revealed no systematic dependence on temperature and practically coincided with the theoretical fractal dimension of the skeleton of a two-dimensional percolation cluster (1.62) [10].

That the V_{DC} oscillatory structure is actually fractal is additionally confirmed by the computations using the rescaled range method, or R/S analysis, also called the Hurst method [13]. Its essence consists in studying the normalized (by the standard deviation S) difference R between the maximum and minimum accumulated deviations of the random value from its mean. As Hurst found out, the observed dependence of the normalized range R/S on the excerpt length L describing this range, for various natural processes is well fitted by a power law $R/S \sim L^{K_H}$ with the exponent K_H close to 0.7. At the same time, if the temporal series are related to random processes with independent values and a finite variance, then $K_H = 0.5$. As shown by Mandelbrot [12] the exponent K_H is related to the fractal dimension D_B in covering the accumulated deviation by the formula

$$D_B = 2 - K_H \quad (2)$$

for self-affine curves.

Fig.6 shows the result of the R/S analysis program applied to an example of voltage fluctuations on a Sn-Ge sample in the vicinity of the superconducting transition. The R/S value is first calculated for the entire excerpt, then the latter is halved, and two independent R/S values are obtained that characterize the half-excerpts separately. Then the number L of points is halved again and again, until it becomes less than 8; at each step the number of independent regions is doubled. The results that correspond to the same value of L are averaged and the R/S values are plotted on the double-logarithmic scale as a function of the number of points L . The exponent K_H is obtained by the least-squares method. In the figure $K_H = 0.84 \pm 0.01$. The broken line shows the dependence at $K_H = 0.5$, i.e. "white"

Figure 5: Determination of cellular fractal dimensions of the $V_{DC}(H)$ curves. The dependence of the number N_c of cells covering the $V_{DC}(H)$ on the cell scale b .

Figure 6: The dependence of the number N_c of cells covering the accumulated deviation of the $V_{DC}(H)$ curve on the cell scale b and diagram of $\log(R/S)$ versus $\log(L)$.

Figure 7: A typical spectrum of voltage oscillation power P in magnetic field. The area a^2 of the quantization contour plays the role of frequency ($1/H$ in our case).

noise. One can see in Fig.6 that $D_B = 1.15 \pm 0.01$, which agrees with expression 2. All these data confirm that the oscillatory structure of $V_{DC}(H)$ is actually fractal.

The observed oscillatory structure is a superposition of quantum oscillations of voltage occurring in Josephson contours of various size up to the geometrical size of the sample. The fractal (i.e., self-similar) behavior of $V_{DC}(H)$ on all the scales in the magnetic field was also confirmed by the Fourier analysis of the $V_{DC}(H)$ dependence. The $V_{DC}(H)$ oscillation intensity spectrum (presented in Fig.7) indicates the absence of clearly manifested peaks, viz., preferred magnetic fields and, hence, quantizing contours.

3.2 Fractal analysis of voltage time series (electric “noise”)

The electric noise and, in particular, the “1/f noise”, observed in percolation systems seems to be determined by the percolation fractal geometry of the systems. Hence, it is reasonable to use the fractal language to the description of random time series and to the search of promising mathematical model of the mechanism of the “1/f noise” origination.

First, we will describe the procedure we used to interpret experimental data.

Fig.8 shows a typical time series (Fig.8a), its energy spectrum (Fig.8b) and the autocorrelated function (Fig.8c). One can see that the fluctuations under study have a sizable correlation on large scales, which narrows the range of feasible mathematical models. Actually, there remain with two ways that permit constructive testing:

- the model of the colored noise with a power-law spectrum and random phases;
- the model of a randomized dynamic system with few degrees of freedom (at least, for the high-frequency part of the spectrum).

Both the hypotheses can be verified by using the same procedure of fractal analysis [14], which is able, as a minimum, to reject one or another model. Let us consider this opportunity in more detail.

In this case under fractal analysis we mean the following sequence of steps:

- projecting the experimental scalar realization into the space of dimension $m = 1, 2, 3, \dots$ (imbedding dimension) by using the Takens procedure [14, 15, 16];
- calculating the correlation integral $C(r, m)$ by using the Grassberger procedure [17] for each projection;

Figure 8: Time series of voltage fluctuations at $T = 3.607K$ (a), its energy spectrum (b) and the autocorrelated function (c) ($t_0 = 85ms$ - counting period).

Figure 9: D_2 dependence on m at $Lf_0 = 200$ for typical realizations registered at various temperatures.

- finding the scaling regions and determining the correlated dimension $D_2(m)$ for each projection;
- investigating the dependence of the correlated dimension D_2 on the imbedding dimension and other parameters.

As is known, in the limit of the infinite realization length L the correlated dimension is determined by the relation

$$D_2(m) = \lim_{L \rightarrow \infty} \lim_{r \rightarrow 0} \frac{d(\log C(r, m))}{d(\log r)} \quad (3)$$

Practically, for a finite realization one determines regions of linear dependence (scaling regions) $\log(C)$ versus $\log(r)$, whose slope yields the estimate of the value D_2 :

$$\log(C(r, m)) = D_2(m) \log(r) + A \quad \text{for } r \text{ in } [r_1 \dots r_2] \quad (4)$$

It was shown [15] that one should expect and account for scaling regions not smaller than one decimal order in $C(r_2)/C(r_1)$, at least, for the imbedding dimension greater than 5. Systematic and random errors in estimating D_2 were evaluated in [15] as well.

It is most essential for the present work that in realizations of finite-dimensional dynamic systems the D_2 dependence on m has a typical shape of a plateau (Fig.9) when $m > D$, and if the plateau is absent, it enables one to reject the conjecture of a dynamic origin of a realization.

On the other hand, applications of this procedure to a realization of the colored noise with the power spectrum and random phases were studied in Refs.[18, 19]. It was shown [19] that the results depended substantially on the ratio of the realization length L and the lowest frequency f_0 of the realization spectrum. Namely, at $Lf_0 < 100$ there is a fictitious plateau on the $D_2(m)$ dependence; the plateau disappears in the opposite case where $Lf_0 > 100$ (the threshold value varies, depending on the exponent of the power spectrum, from one to several hundreds). The physical sense of this phenomenon is related to the manifestation of the signal non-stationarity at $Lf_0 < 100$. Thus, the colored noise conjecture can also be, as a minimum, rejected, if the D_2 dependence on Lf_0 behaves not in the described fashion, namely when the plateau at $Lf_0 > 100$ disappears. This analysis is similar to checking the “null hypothesis”, which is mentioned in the literature [19].

Figure 10: Dependence of the correlated dimension D_2 on the imbedding dimension m at $T = 3.607K$ on varying the parameter Lf_0 .

In our case, the realization energy spectrum is obviously a power-law one, all the way down to zero frequency. Varying the parameter Lf_0 can be done by filtering realizations with a symmetric non-recursive filter which gives way only to the high-frequency spectrum part $f > f_0$. So in this paper we investigate filtered realizations of the initial set by applying the fractal analysis. In each realization we analyze the dependence of the correlated dimension D_2 on

- the imbedding dimension m at a fixed parameter Lf_0 ,
- the parameter Lf_0 at a fixed imbedding dimension m .

Here we shall describe and analyze some results of the above-described procedure to the experimentally obtained realizations of voltage fluctuations registered at various temperatures near and below the mean point of the superconducting transition.

In all realizations the imbedding dimension m varied within the range 1..20, while the non-stationarity parameter Lf_0 took on values 1, 40, 100, 200, 400 at the realization length of 8100 points. The separation between elements of one vector in the Takens procedure was taken equal to the position of the first zero of the autocorrelated function, while in the Grassberger procedure vectors were excluded if the time separation between them was less than this value; this was done to exclude the so-called “shoulder” effect [20]. Fig.9 shows a plot of the D_2 dependence on m at $Lf_0 = 200$ for typical realizations registered at various temperatures. As seen in the figure, the dependence has a pronounced plateau at rather moderate values of D_2 . This confirms the reliable registration of the non-random nature of the realization under study. In other words, the conjecture of the dynamic nature of the studied realizations cannot, as a minimum, be rejected.

An additional confirmation of the latter statement is provided in Fig.10, which demonstrates the change in the $D_2(m)$ dependence on varying the parameter Lf_0 . (Here we have the same realization $T = 3.607K$ which was presented in Fig.8a). One can see that at $Lf_0 > 100$ the plateau displays no tendency to disappearing. Thus, the result of the analysis enables us to reject the hypothesis of colored noise with a power-law spectrum and random phases.

The only circumstance that remains unexplained is the lowering of the plateau on the $D_2(m)$ dependence when the parameter Lf_0 increases beyond the threshold value $Lf_0 = 100$.

In principle, in the spirit of Ref.[21], one could attempt, to interpret such a behavior as a mixture of several dynamic systems with different characteristic fre-

quencies, whose relative weights vary with changing the threshold frequency of the filter f_0 . We believe, however, that in this case such an interpretation would be too arbitrary. That is why we have attempted to interrelate spatial and temporal dependences typical just for this case. We suggest a simple dynamic model which accounts for our experimental data without loss of any details.

4 A model of “1/f noise” generation in a percolation system.

In the preceding sections, we have presented results of investigating the fractal nature of the dependences $V_{DC}(H)$, and analyzed the results of applying the fractal analysis to experimental time realizations of voltage fluctuations in superconducting percolation films near the superconducting transition temperature. In particular, we have presented the arguments for inadequacy of the colored random noise and for the adequacy of the deterministic dynamic model in a stochastic mode. However, the correlated dimension dependence on the filtration parameter Lf_0 has remained unclear.

In the percolation system under study, another, better studied, phenomenon is observed which has a fractal nature and can have a relation to the investigated fluctuations. The fractal nature of the dependence $V_{DC}(H)$ was investigated in detail above via the calculation of its cell dimension and the R/S analysis, as well as by direct scale transformations. Since the internal cell dimension of the plot of $V_{DC}(H)$ tends to 1.6-1.7, while that of the accumulated signal tends to the stationary value near 1.15, we can conclude that the plot of $V_{DC}(H)$ is close to the pointwise approximation of the fractal curve derivative.

Suppose that in the sample there are oscillations of a magnetic field, of internal or external origin (Fig.11c). Then the passive detection of these oscillations with the fractal transition function of detector $V_{DC}(H)$ (Fig.11a) will contribute to the voltage across the sample (Fig.11b). Here two important peculiarities will be observed. First, since the fractal curve derivative can have arbitrarily large slope angles (to within the accuracy owing to its finite approximation determined by the geometric dimensions of the sample), any signal, however weak, can be amplified to the maximum value while detecting it. Second, the multi-scale pattern of the fractal curve will impose a power-law spectrum on very simple oscillations detected.

In order to verify this model, we repeated the fractal analysis procedure for a realization simulated as follows. First of all we modeled the transition function $V_{DC}(H)$, using the finite-difference derivative of the Weierstrass function $W(x)$, obtained by summing a finite number K of periodic terms.

$$V_{sim}(H) = W(H + \Delta) - W(H) \quad (5)$$

$$W(x, K) = \sum_{i=1}^K 2^{-\alpha i} \cos(2^{ix}) \quad (6)$$

Then the model realization for voltage time fluctuations $E(t)$ has the form

$$E(t) = V_{sim}(h(t)), \quad (7)$$

where $h(t)$ is the model of magnetic field oscillations of any kind. For better visualization we tried the simplest functions

$$h(t) = \sin(t), \quad (8)$$

Figure 11: Passive transformation of magnetic field oscillations to electric “1/f noise”.

Figure 12: The energy spectrum of a model realization.

$$h(t) = \sin(t) + \sin\left(\frac{\pi t}{4}\right). \quad (9)$$

Here the parameter α was selected so that both the energy spectrum of the realization $E(t)$, and the R/S parameter of the transition function $V_{sim}(H)$, as well as the cell dimension of accumulated signal would coincide with the values calculated for one of the experimental realizations. Both coincidences were reached by varying only the parameter α ; here the dependence on Δ and number K of terms of the trigonometric series appeared insignificant. Practically, we used the following values: $\alpha = 1.5$, $\Delta = 10^{-4}$, $K = 10$. Below we present the results of analysis of a model realization with an external signal in the form of Eq.9. Here the energy spectrum of the model realization (see Fig.12) has the form quite similar to the natural data.

In Figs.13a and 13b the $D_2(m)$ dependence is shown for various values of the parameter Lf_0 . One can see that the dependence has all the characteristic features of the experimental realizations shown in Fig.10. Namely, there are well-shaped

Figure 13: D_2 dependence on m for the model realization at $Lf_0 = 100$ (a), $Lf_0 = 200$ (b).

plateaux on the dependences $D_2(m)$, and the plateaux move down with the growth of the parameter Lf_0 . As in the experimental case, the plateaux display no tendency to disappear when $Lf_0 > 100$.

All this enables us to assert that the suggested model has passed the verification by the fractal analysis procedure, and the model reproduces qualitatively all important peculiarities. Besides, the model has an important property: it is not arbitrary in the physical system studied.

5 Conclusion

In this paper we have made a constructive attempt to confirm or to reject various models for describing voltage fluctuations in Sn-Ge thin-film superconducting composites in the vicinity of the percolation threshold at temperatures close to the superconducting transition. We did our best to confine ourselves to studying experimental realizations and to abstain from speculative theorizing.

We have employed the procedure for fractal analysis. The procedure enables us, as a minimum, to reject two hypotheses about the nature of the realizations observed: the model of a determinate dynamic system and the model of the colored noise with a power-law energy spectrum and random phases. If one adds to this list obviously inadequate models of the Markov process, or the process of a small correlation radius, and that of a determinate system with a simple regular behavior, then one obtains a complete list of models which can be tested by using constructive procedures.

We believe that our results permit us to reject the colored noise model reliably. However, the alternative model of a dynamic process is not quite adequate, since it does not account for the correlated dimension change in the function of the processing parameters. This demands either a more or less arbitrary complication of the dynamic model or a complete change of the viewpoint. We have chosen the latter and, retaining the simple dynamic nature, suggested another model of the process observed. We believe that our model successfully interrelates various physical phenomena typical for the studied system.

We have suggested and verified the model of the voltage fluctuation origin by a passive transformation of any magnetic field oscillations with the transformation fractal function. We have studied in detail the fractal nature of the transition function of such a transforming mechanism for the Sn-Ge percolation system. Its fractality explains both the amplification of arbitrarily weak signals to the observable level and a universal spectrum composition of realizations, even for the simplest detectable processes. We have shown that the corresponding model realizations manifest all typical peculiarities of the dependence of the correlated dimension D_2 on the dimension of the imbedding and the stationarity parameter Lf_0 .

The fractal character of the transition function seems to be related to the existence of a wide and self-similar distribution of Josephson contour areas. Some theoretical models of this phenomenon were reported by Grib [22]. The formation of a transition function having a fractal symmetry seems to be typical for a broad class of percolation systems (such are, for example, all the objects at the point of the second order phase transition). From this point of view the suggested model can be regarded as one of universal models of generating the “1/f noise” in a large series of physical as also non-physical objects and to confirm the fundamental fractal nature of the “1/f noise”. Here we have not discussed the variation profile of the fractal dimension of the transition function $V_{DC}(H)$ and the correlated dimensions D_2 with the variation of temperature. A separate paper will be devoted to this question. This does not pretend to exhaust the topic. On the contrary, we intend to start and stimulate a discussion of the phenomenon.

6 Acknowledgments

This work was partially supported by the Fundamental Research Foundation of the Ukrainian State Committee on Science and Technology.

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